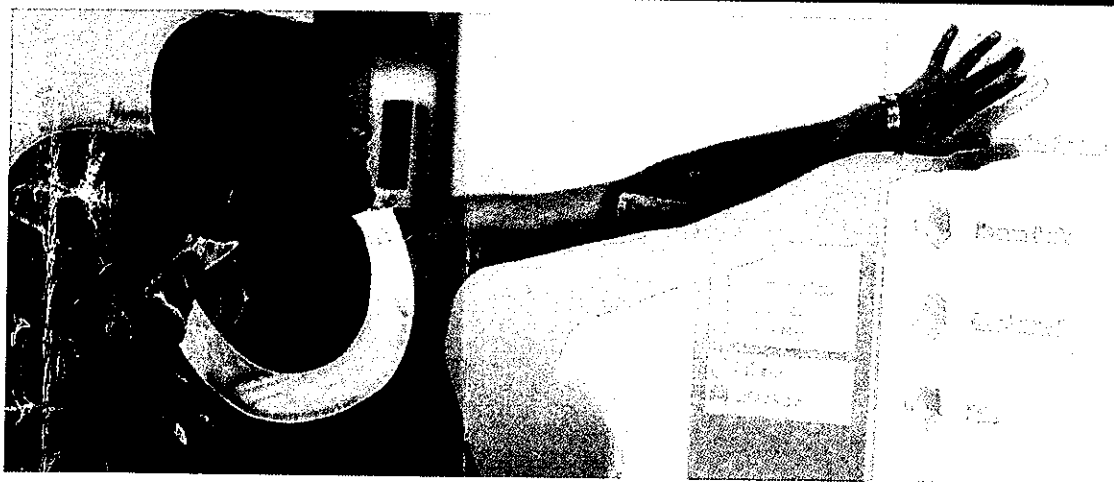
# Mathematics ABC Assessment Workshop

2009



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# Contents

# Designing an Assessment Instrument — exam / assignment, etc

In designing an assessment instrument the following procedure will allow the construction of the instrument in which desired aspects of the general objectives will be identifiable, and these aspects will be associated with the appropriate descriptors of the standards matrix. The associated instrument-specific criteria sheet will inform students of the specific requirements of the instrument and will provide simple feedback on student performance matched to the standards descriptors.

1. **Start with the General Objectives**
2. **Determine what aspects of the General Objectives you wish to assess.**
3. **Refer to the Standards Matrix for the relevant descriptors for the aspects of the General Objectives chosen**
4. **Develop an instrument or instruments that will allow students to demonstrate these attributes**
5. **Develop an instrument-specific criteria sheet using the chosen attributes of standards matrix as a basis**

## 1. Start with the General Objectives

For Mathematics A, the assessable General Objectives are:

### Knowledge and Procedures:

- manipulate simple rules and formulas
- access, select and apply rules and formulas
- recall, select and apply mathematical procedures to situations that are similar to situations already encountered
- apply a sequence of mathematical procedures in situations that are similar to situations already encountered
- use mathematical technology and geometrical instruments.

### Modelling and problem solving:

- interpret, clarify and analyse problems
- use strategies to model and solve problems
- investigate alternative solutions and/or procedures to problems
- make decisions informed by mathematical reasoning
- reflect on the effectiveness of mathematical models, including the recognition of strengths and limitations.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop logical sequences within a response expressed in everyday language, mathematical language, or a combination of both, as required, to justify conclusions, solutions or propositions
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

For Mathematics B, the assessable General Objectives are:

## Knowledge and procedures

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology.

## Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.

## Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

For Mathematics C the assessable General Objectives are:

## Knowledge and procedures

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology
- demonstrate knowledge and use of the nature of mathematical proof.

## Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- modify mathematical models as appropriate.

## Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results

- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both
- provide supporting arguments in the form of a proof and recognise that a proof may require more than a verification of a number of instances.

## 2. Determine what aspects of the General Objectives you wish to assess.

For Mathematics A, the general objectives chosen may be:

### Knowledge and Procedures:

- manipulate simple rules and formulas
- access, select and apply rules and formulas
- recall, select and apply mathematical procedures to situations that are similar to situations already encountered
- apply a sequence of mathematical procedures in situations that are similar to situations already encountered
- use mathematical technology and geometrical instruments.

Modelling + Problem Solving

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop logical sequences within a response expressed in everyday language, mathematical language, or a combination of both; as required, to justify conclusions, solutions or propositions
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

For Mathematics B, the general objectives chosen may be:

## Knowledge and procedures

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology.

## Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

For Mathematics C, the general objectives chosen may be:

## Knowledge and procedures

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology
- demonstrate knowledge and use of the nature of mathematical proof.

## Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both
- provide supporting arguments in the form of a proof and recognise that a proof may require more than a verification of a number of instances.

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

## 3. Refer to the Standards Matrix for the relevant descriptors for the aspects of the General Objectives chosen

For Mathematics A, the attributes of the Knowledge and procedures and Communication and justification criteria in the standards descriptors corresponding to the chosen aspects of the general objectives would be:

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>• accurate use of rules and formulas in simple through to complex situations | The student's work has the following characteristics:<br>• accurate use of rules and formulas in simple situations or use of rules and formulas in complex situations | The student's work has the following characteristics:<br>• use of rules and formulas in simple routine situations | The student's work has the following characteristics:<br>• use of given rules and formulas in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given rules and formulas in simple rehearsed situations |
| | • application of simple through to complex sequences of mathematical procedures in routine and non-routine situations | • application of simple sequences of mathematical procedures in non-routine situations or complex sequences in routine situations | • application of simple sequences of mathematical procedures in routine situations | • application of simple mathematical procedures in simple rehearsed situations | • attempted use of simple mathematical procedures in simple rehearsed situations |
| | • appropriate selection and accurate use of technology | • appropriate selection and accurate use of technology | • selection and use of technology | • use of technology | • attempted use of technology |

| Criterion | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• accurate and appropriate use of mathematical terminology and conventions in simple non-routine through to complex routine situations | The student's work has the following characteristics:<br>• accurate and appropriate use of mathematical terminology and conventions in simple non-routine and/or complex routine situations | The student's work has the following characteristics:<br>• appropriate use of mathematical terminology and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology and conventions in simple rehearsed situations | The student's work has the following characteristics:<br>• use of mathematical terminology or conventions in simple rehearsed situations |
| | • organisation and presentation of information in a variety of representations in simple non-routine through to complex routine situations | • organisation and presentation of information in a variety of representations in simple non-routine and/or complex routine situations | • organisation and presentation of information in a variety of representations in simple routine situations | • presentation of information in simple rehearsed situations | |
| | • analysis and translation of information displayed from one representation to another in complex routine situations | • analysis and translation of information displayed from one representation to another in simple routine situations | • translation of information displayed from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop logical sequences in simple non-routine through to complex routine situations using everyday and/or mathematical language | • use of mathematical reasoning to develop logical sequences in simple non-routine and/or complex routine situations using everyday and/or mathematical language | • development of logical sequences in simple routine situations using everyday and/or mathematical language | | |
| | • justification of the reasonableness of results obtained through technology or other means | | | | |

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

NOTE - must be done for every single assessment task, whether exam, prac or assignment!

For Mathematics B, the attributes of the Knowledge and procedures and Communication and justification criteria in the standards descriptors corresponding to the chosen aspects of the general objectives would be:

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations | The student work has the following characteristics:<br>• use of stated rules and procedures in simple situations | The student work has the following characteristics:<br>• statements of relevant mathematical facts |
| | • application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations | • application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations | • application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations | | |
| | • numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | • numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations | • numerical calculations, spatial sense and algebraic facility in routine, simple life-related or abstract situations | • numerical sense, spatial sense and/or algebraic facility in routine or simple tasks | |
| | • appropriate selection and accurate use of technology | • appropriate selection and accurate use of technology | • selection and use of technology | • use of technology | • use of technology |

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions in simple or routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions |
| | • organisation and presentation of information in a variety of representations | • organisation and presentation of information in a variety of representations | • organisation and presentation of information | • presentation of information | • presentation of information |
| | • analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine | • analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine | • translation of information from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language | • use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | | |
| | • coherent, concise and logical justification of procedures, decisions and results | • coherent and logical justification of procedures, decisions and results | • justification of procedures, decisions or results | | |
| | • justification of the reasonableness of results | | | | |

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

For Mathematics C, the attributes of the Knowledge and procedures and Communication and justification criteria in the standards descriptors corresponding to the chosen aspects of the general objectives would be:

| *Criterion* | *Standard A* | *Standard B* | *Standard C* | *Standard D* | *Standard E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks in life-related and abstract situations | The student work has the following characteristics:<br>• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations | The student work has the following characteristics:<br>• use of stated rules and procedures in simple situations | The student work has the following characteristics:<br>• statements of relevant mathematical facts |
| | • application of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | • application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations | • application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations | | |
| | • numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations | • numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations | • numerical sense, spatial sense and algebraic facility in routine, simple life-related or abstract situations | • numerical sense, spatial sense and/or algebraic facility in routine or simple tasks | |
| | • appropriate selection and accurate use of technology | • appropriate selection and accurate use of technology | • selection and use of technology | • use of technology | • use of technology |
| | • knowledge of the nature of and use of mathematical proof | | | | |

| | *Standard A* | *Standard B* | *Standard C* | *Standard D* | *Standard E* |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions in simple or routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions |
| | • organisation and presentation of information in a variety of representations | • organisation and presentation of information in a variety of representations | • organisation and presentation of information | • presentation of information | • presentation of information |
| | • analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine | • analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine | • translation of information from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language | • use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | | |
| | • coherent, concise and logical justification of procedures, decisions and results | • coherent and logical justification of procedures, decisions and results | • justification of procedures, decisions or results | | |
| | • justification of the reasonableness of results | | | | |
| | • provision of supporting arguments in the form of proof | | | | |

Please note here that we are only considering the Knowledge and procedures and Communication and justification criteria in this exercise.

## 4. Develop an instrument or instruments that will allow students to demonstrate these attributes

For the particular topic you are wanting to assessment, gather/generate assessment items that will satisfy the requirements as indicated in the particular attributes that have been chosen from the standards matrix. This does not mean that traditionally used items have to be rejected – many are still valid. However care should be exercised when assessment items are developed to make sure that the aspects of the chosen attributes will be met by the items.

For Mathematics A, the following is presented as an example of items that could be presented that will satisfy the attributes identified in the standards matrix. The Modelling and problem solving criteria is not being considered in this assessment instrument. The particular attributes for each standard are included for completeness only and would not need to be included with a normal supervised assessment.

**Question 1 (Standard D – application of simple mathematical procedures in simple rehearsed situation)**

Using the formula

Markup = marked price – cost price

Calculate the cost price of an article which had a marked price of $280 and a markup of $26.

**Question 2 (Standard C – application of simple sequences of mathematical procedures in routine situations)**

(i) An aluminium dinghy that cost a boatyard $1200 to build is sold for $2160. What is the percentage markup?

(ii) A hardware shop uses a 65% markup. What would the shop charge for a spade with the cost price of $12?

(iii) Calculators are sold to shops at a trade discount of 40%. What do the shops pay for a calculator with a list price of $45?

(iv) A sales representative bought a new car for $32 500 and after using it for 2 years sold it for $18 600. What was the percentage loss?

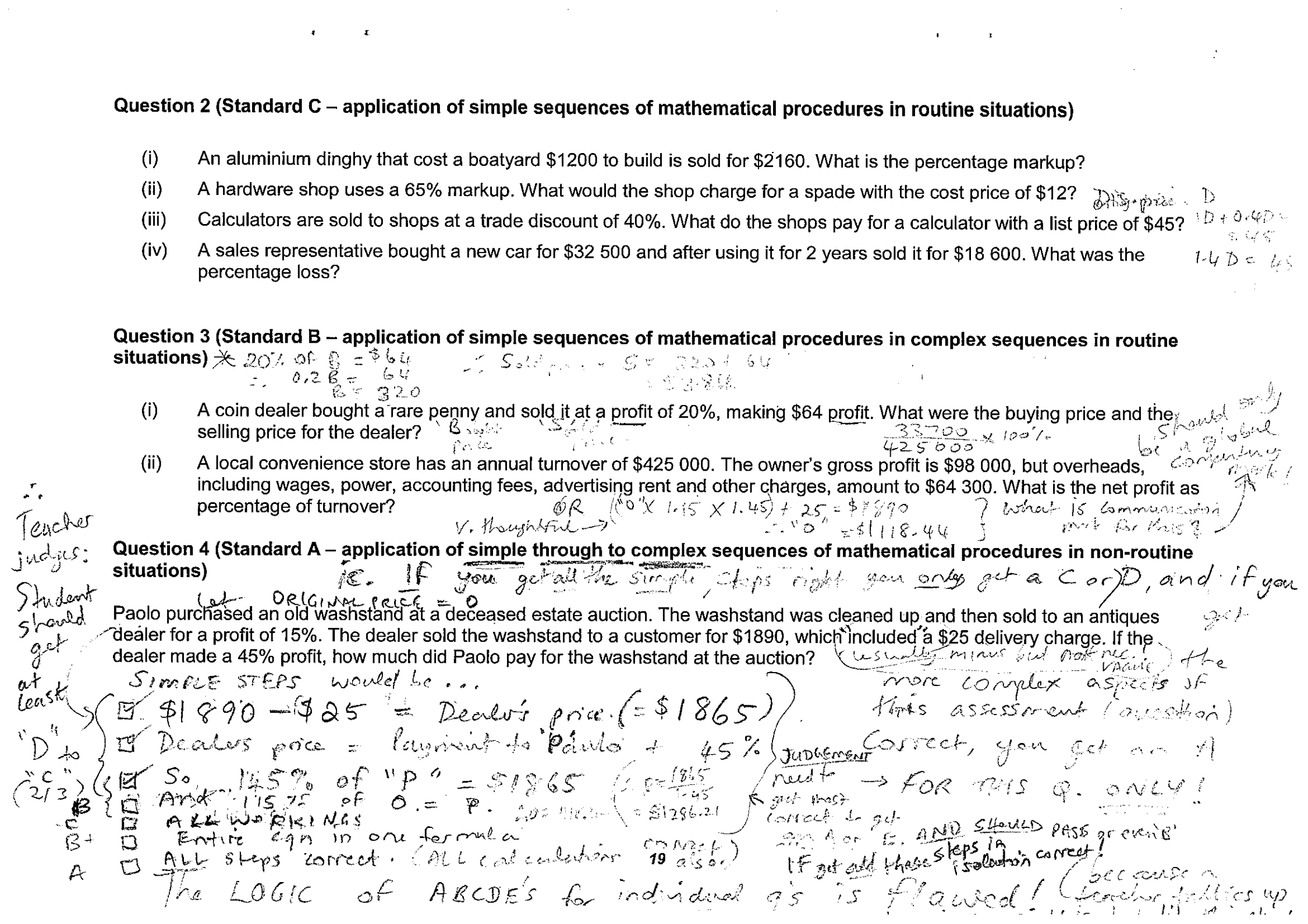
**Question 3 (Standard B – application of simple sequences of mathematical procedures in complex sequences in routine situations)**

(i) A coin dealer bought a rare penny and sold it at a profit of 20%, making $64 profit. What were the buying price and the selling price for the dealer?

(ii) A local convenience store has an annual turnover of $425 000. The owner's gross profit is $98 000, but overheads, including wages, power, accounting fees, advertising rent and other charges, amount to $64 300. What is the net profit as percentage of turnover?

**Question 4 (Standard A – application of simple through to complex sequences of mathematical procedures in non-routine situations)**

Paolo purchased an old washstand at a deceased estate auction. The washstand was cleaned up and then sold to an antiques dealer for a profit of 15%. The dealer sold the washstand to a customer for $1890, which included a $25 delivery charge. If the dealer made a 45% profit, how much did Paolo pay for the washstand at the auction?



For Mathematics B, the following is presented as an example of items that could be presented that will satisfy the attributes identified in the standards matrix. Note that because only application of rules and definitions is being considered, there is no item which references the D standard because no "stated rule" is being considered. The Modelling and problem solving criteria is not being considered in this assessment task. The particular attributes for each standard are included for completeness only and would not need to be included with a normal supervised assessment.

**Question 1 (Standard C - application of mathematical definitions, rules and procedures in routine, simple abstract situations)**

Solve the following equations for $x$, given that the values lie in the range $0 \leq x \leq 2\pi$ .

(i) $\cos x = -0.5$

(ii) $2\sin x = \sqrt{3}$

(iii) $\tan x = 1$

(iv) $2 + 2\sin x = 1$

**Question 2 (Standard B - application of mathematical definitions, rules and procedures in routine complex tasks, in abstract situations)**

Solve the following equations for $x$, given that the values lie in the range $0 \leq x \leq 2\pi$ .

(i) $2\cos\left(x + \frac{\pi}{6}\right) + 3 = 4$

(ii) $2\sin 3x = -\sqrt{2}$

(iii) $\tan^2 x - 4\tan x = 0$

**Question 3 (Standard A - application of mathematical definitions, rules and procedures in non- routine complex tasks, in abstract situations)**

Solve the following equations for $x$, given that the values lie in the range $-\pi \leq x \leq \pi$.

(i) $\tan^2 x + 3\tan x - 4 = 0$

(ii) $6\cos^2 x + 5\sin x = 7$

For Mathematics C, the following is presented as an example of items that could be presented that will satisfy the attributes identified in the standards matrix. Note that because only application of rules and definitions is being considered, there is no item which references the D standard because no "stated rule" is being considered. The Modelling and problem solving criteria is not being considered in this assessment task. The particular attributes for each standard are included for completeness only and would not need to be included with a normal supervised assessment.

**Question 1 (Standard C - application of mathematical definitions, rules and procedures in routine, simple abstract situations)**

(a) Find the value of $\sec\theta$ if $\tan\theta$ has the value of 4.2

(b) If $\cos\theta = 0.8$, find values for $\cot\theta$.

(c) Express $\cot\theta$ in terms of $\sec\theta$.

(d) Use an expansion formula to show that $\sin 2A = 2\sin A\cos A$

**Question 2 (Standard B - application of mathematical definitions, rules and procedures in routine complex tasks, in abstract situations)**

(a) Use an expansion formula to show that $\cos 3A = 4\cos^3 A - 3\cos A$

(b) Prove the identity $\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)=\frac{2}{\cos^2 x-\sin^2 x}$

(c) Express the combined function $\sin x - 3\cos x$ as a single cosine function.

**Question 3 (Standard A - application of mathematical definitions, rules and procedures in non- routine complex tasks, in abstract situations)**

(a) Solve $3\sin x + \sqrt{3}\cos x = 3$ for $0 \le x \le 2\pi$

(b) Find an exact value for $\tan 255^0$

## 5. Develop an instrument-specific criteria sheet using the chosen attributes of standards matrix as a basis

For the Mathematics A assessment instrument developed, the instrument-specific criteria sheet could have the following format:

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristic:<br>• application of complex sequences of mathematical procedures in non-routine finance situations | The student's work has the following characteristic:<br>• application of simple sequences of mathematical procedures in complex sequences in routine finance situations | The student's work has the following characteristic:<br>• application of simple sequences of mathematical procedures in routine finance situations | The student's work has the following characteristic:<br>• application of simple mathematical procedures in simple rehearsed finance situations | The student's work has the following characteristic:<br>• attempted use of simple mathematical procedures in simple rehearsed finance situations |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine finance situations | • accurate and appropriate use of mathematical terminology and conventions in complex routine finance situations | • appropriate use of mathematical terminology and conventions in simple routine finance situations | • use of mathematical terminology and conventions in simple rehearsed finance situations | • use of mathematical terminology or conventions in simple rehearsed finance situations |

Indicate on the criteria sheet the highest standard that the student has achieved within the particular attribute of those chosen from the standards matrix.

For the Mathematics B assessment instrument developed, the instrument-specific criteria sheet could have the following format:

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics:<br>• application of mathematical definitions, rules and procedures in non-routine complex tasks, in abstract situations involving trigonometry | The student work has the following characteristics:<br>• application of mathematical definitions, rules and procedures in routine complex tasks, in abstract situations involving trigonometry | The student work has the following characteristics:<br>• application of mathematical definitions, rules and procedures in routine, abstract situations involving trigonometry | The student work has the following characteristics: | The student work has the following characteristics: |
| *Communication and justification* | • appropriate interpretation and use of mathematical terminology, symbols and conventions in non-routine complex, in abstract situations involving trigonometry<br>• use of mathematical reasoning to develop coherent, concise and logical sequences within a response in complex abstract situations using everyday and mathematical language | • appropriate interpretation and use of mathematical terminology, symbols and conventions in routine complex, in abstract situations involving trigonometry<br>• use of mathematical reasoning to develop coherent and logical sequences within a response in simple abstract situations using everyday and/or mathematical language | • appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations involving trigonometry<br>• use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | • use of mathematical terminology, symbols or conventions in simple situations involving trigonometry<br>• | • use of mathematical terminology, symbols or conventions<br>• |

For the Mathematics C assessment instrument developed, the instrument-specific criteria sheet could have the following format:

| *Criterion* | *Standard A* | *Standard B* | *Standard C* | *Standard D* | *Standard E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics: | The student work has the following characteristics: | The student work has the following characteristics: | The student work has the following characteristics: | The student work has the following characteristics: |
| | • application of mathematical definitions, rules and procedures in non-routine complex tasks, in abstract situations involving advanced trigonometry | • application of mathematical definitions, rules and procedures in routine complex tasks, in abstract situations involving advanced trigonometry | • application of mathematical definitions, rules and procedures in routine, simple abstract situations involving advanced trigonometry | | |
| *Communication and justification* | • appropriate interpretation and use of mathematical terminology, symbols and conventions in complex non-routine, in abstract situations involving advanced trigonometry | • appropriate interpretation and use of mathematical terminology, symbols and conventions in complex routine tasks in abstract situations involving advanced trigonometry | • appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations involving advanced trigonometry | • use of mathematical terminology, symbols or conventions in simple situations involving advanced trigonometry | • use of mathematical terminology, symbols or conventions |
| | • use of mathematical reasoning to develop coherent, concise and logical sequences within a response in complex and in abstract situations using everyday and mathematical language | • use of mathematical reasoning to develop coherent and logical sequences within a response in complex and in abstract situations using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | | |

# Modelling and problem solving
# Mathematics A

## 1. Start with the General Objectives (We are only considering the Modelling and problem solving and Communication and justification general objectives in the exercise)

For Mathematics A, the assessable General Objectives are:

### Modelling and problem solving:

- interpret, clarify and analyse problems
- use strategies to model and solve problems
- investigate alternative solutions and/or procedures to problems
- make decisions informed by mathematical reasoning
- reflect on the effectiveness of mathematical models, including the recognition of strengths and limitations.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop logical sequences within a response expressed in everyday language, mathematical language, or a combination of both, as required, to justify conclusions, solutions or propositions
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 2. Determine what aspects of the General Objectives you wish to assess.

### Modelling and problem solving:

- interpret, clarify and analyse problems
- use strategies to model and solve problems
- investigate alternative solutions and/or procedures to problems
- make decisions informed by mathematical reasoning
- reflect on the effectiveness of mathematical models, including the recognition of strengths and limitations.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop logical sequences within a response expressed in everyday language, mathematical language, or a combination of both, as required, to justify conclusions, solutions or propositions
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 3. Refer to the Standards Matrix for the relevant descriptors for the aspects of the General Objectives chosen

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student's work has the following characteristics:<br>• use of strategies to model and solve problems in complex routine through to simple non-routine situations | The student's work has the following characteristics:<br>• use of strategies to model and solve problems in routine through to simple non-routine situations | The student's work has the following characteristics:<br>• use of familiar strategies for problem solving in simple routine situations | The student's work has the following characteristics:<br>• use of given strategies for problem solving in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given strategies for problem solving in well-rehearsed situations |
| | • investigation of alternative solutions and/or procedures to complex routine through to simple non-routine problems | • investigation of alternative solutions and/or procedures to routine problems | | | |
| | • informed decisions based on mathematical reasoning in complex routine through to simple non-routine situations | • informed decisions based on mathematical reasoning in routine situations | • informed decisions based on mathematical reasoning in simple routine situations | | |
| | • reflection on the effectiveness of mathematical models including recognition of the strengths and limitations of the model | • recognition of the strengths and limitations of the model in simple situations | | | |

| *Criterion* | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• accurate and appropriate use of mathematical terminology and conventions in simple non-routine through to complex routine situations | The student's work has the following characteristics:<br>• accurate and appropriate use of mathematical terminology and conventions in simple non-routine and/or complex routine situations | The student's work has the following characteristics:<br>• appropriate use of mathematical terminology and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology and conventions in simple rehearsed situations | The student's work has the following characteristics:<br>• use of mathematical terminology or conventions in simple rehearsed situations |
| | • organisation and presentation of information in a variety of representations in simple non-routine through to complex routine situations | • organisation and presentation of information in a variety of representations in simple non-routine and/or complex routine situations | • organisation and presentation of information in a variety of representations in simple routine situations | • presentation of information in simple rehearsed situations | |
| | • analysis and translation of information displayed from one representation to another in complex routine situations | • analysis and translation of information displayed from one representation to another in simple routine situations | • translation of information displayed from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop logical sequences in simple non-routine through to complex routine situations using everyday and/or mathematical language | • use of mathematical reasoning to develop logical sequences in simple non-routine and/or complex routine situations using everyday and/or mathematical language | • development of logical sequences in simple routine situations using everyday and/or mathematical language | | |
| | • justification of the reasonableness of results obtained through technology or other means | | | | |

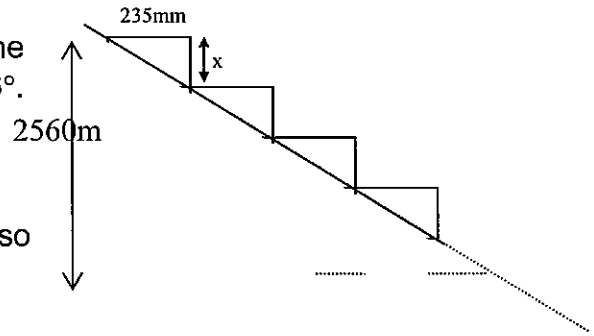
## 4. Develop an instrument or instruments that will allow students to demonstrate these attributes

### Question (Informed decision making)

Local shire council zoning regulations state that any development on canals and waterways is restricted to a height of 30m. The local environmental committee is concerned that an apartment block under construction breaches this law. In order to get an approximate estimate of the height of this building, a measurement is taken from the opposite edge of the canal and the angle of elevation to the top of the building is found to be 32°. Another reading is taken 25m back (in a straight line) from the canal edge and the angle of elevation to the top of the building is this time recorded as 21°. The surveyor taking the readings is 1.7m tall. Does the height of the building breach the building code?

### Question

The diagram represents a staircase with treads 235mm wide. This is referred to in the building industry as the "go". The angle the staircase makes with the horizontal is 36°. What is the rise of the stairs (marked x) to the nearest mm?




The building code refers to something called the Slope Relation. This relation is represented by the formula 2R + G where R is the rise and G is the go. This code also requires that $550 < 2R + G < 700$.

Does this staircase satisfy these requirements for the Slope Relation?

If the height of the steps at their highest point is 2560mm, how many steps would you plan for in the staircase? Justify your answer.

## 5. Develop a instrument-specific criteria sheet using the chosen attributes of standards matrix as a basis

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student's work has the following characteristics:<br>• use of strategies to model and solve problems in simple non-routine situations | The student's work has the following characteristics:<br>• use of strategies to model and solve problems in routine situations | The student's work has the following characteristics:<br>• use of familiar strategies for problem solving in simple routine situations | The student's work has the following characteristics:<br>• use of given strategies for problem solving in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given strategies for problem solving in well-rehearsed situations |
| | • informed decisions based on mathematical reasoning in simple non-routine situations | • informed decisions based on mathematical reasoning in routine situations | • informed decisions based on mathematical reasoning in simple routine situations | | |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • accurate and appropriate use of mathematical terminology and conventions in complex routine situations | • appropriate use of mathematical terminology and conventions in simple routine situations | • use of mathematical terminology and conventions in simple rehearsed situations | • use of mathematical terminology or conventions in simple rehearsed situations |

# Modelling and problem solving
# Mathematics B

## 1. Start with the General Objectives

For Mathematics B, the assessable General Objectives are:

### Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 2. Determine what aspects of the General Objectives you wish to assess.

### Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 3. Refer to the Standards Matrix for the relevant descriptors for the aspects of the General Objectives chosen

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple tasks through to routine complex tasks in life-related or abstract situations | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations | The student work has the following characteristics:<br>• evidence of simple problem-solving strategies in the context of problems | The student work has the following characteristic:<br>• evidence of simple mathematical procedures |
| | • identification of assumptions and their associated effects, parameters and/or variables | • identification of assumptions, parameters and/or variables | | | |
| | • use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations | • use of data to synthesise mathematical models in simple situations and generation of data from mathematical models in simple through to complex situations | • use of mathematical models to represent routine, simple situations and generate data | • use of given simple mathematical models to generate data | |
| | • investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems; the strengths and limitations of models, both given and developed | • interpretation of results in the context of simple through to complex problems and mathematical models | • interpretation of results in the context of routine, simple problems | | |

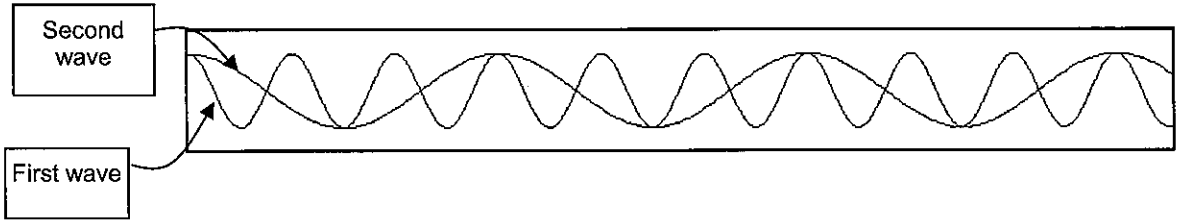
| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions in simple or routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions |
| | • organisation and presentation of information in a variety of representations | • organisation and presentation of information in a variety of representations | • organisation and presentation of information | • presentation of information | • presentation of information |
| | • analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine | • analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine | • translation of information from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language | • use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | | |
| | • coherent, concise and logical justification of procedures, decisions and results | • coherent and logical justification of procedures, decisions and results | • justification of procedures, decisions or results | | |
| | • justification of the reasonableness of results | | | | |

## 4. Develop an instrument or instruments that will allow students to demonstrate these attributes

The following is presented to give an indication of how a standard textbook question can be easily modified to include one of the higher order attributes required for the awarding of an A standard in Modelling and problem solving. It is not imperative that the question chosen should be one of high complexity and be necessarily non-routine to include these higher order attributes.

### Question 1

Sound waves produced by synthesisers follow sinusoidal patterns and drone may be produced by constructive interference of 2 sound waves when they "peak" simultaneously as shown in the diagram below.




If the first wave is modelled by $y = 3\cos 2\left(t - \frac{\pi}{6}\right)$, give three equations that may model the second wave if the drone were to occur every 3 beats of the first as indicated?

**Question 2**

After insulating material has been installed into the ceiling cavity of a house, the temperature, measured in $^0$C, inside the house at $t$ hours after 4 am is given by the expression

$$21-3\cos\left(\frac{\pi t}{12}\right) \text{ for } 0 \le t \le 24$$

and the temperature outside the same house at the same time is given by

$$22-5\cos\left(\frac{\pi t}{12}\right) \text{ for } 0 \le t \le 24$$

Over what period of time is the temperature of the inside of the house less than the temperature of the outside?

**Question 2 (Modified for evaluation of the validity of an argument)**

After insulating material has been installed into the ceiling cavity of a house, the temperature, measured in $^0$C, inside the house at $t$ hours after 4 am is given by the expression

$$21-3\cos\left(\frac{\pi t}{12}\right) \text{ for } 0 \le t \le 24$$

and the temperature outside the same house at the same time is given by

$$22-5\cos\left(\frac{\pi t}{12}\right) \text{ for } 0 \le t \le 24$$

The manufacturer of the insulating material claims that the inside temperature of the house will be less than the temperature outside for 75% of the time during a 24-hour period. Is the manufacturer justified in making this claim?

**Question 3 (Identification of assumptions and their associated effects)**

The temperature, $T^0$C, in an electric oven being heated over a time period, $t$ in minutes, can be represented by the following data table:

| Time (in Minutes) | 2 | 4 | 9 |
|---|---|---|---|
| Temperature (in degrees Celcius) | 80 | 145 | 250 |

The oven is thermostatically controlled so that the oven has a maximum temperature possible of 350 $^0$C. If it is assumed that the change in temperature over time can be modelled by a quadratic function, when will the temperature of the oven reach 300$^0$C?

Discuss your model in relation to this result.

If however, the change in temperature over time is modelled using a logarithmic function, what effect will this have on your calculations?

## 5. Develop an instrument-specific criteria sheet using the chosen attributes of standards matrix as a basis

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student work has the following characteristics: | The student work has the following characteristics:<br>• use of problem-solving strategies to analyse problems to develop responses to non-routine simple tasks in life-related functions | The student work has the following characteristics:<br>• use of problem-solving strategies to develop responses to routine, simple problems in life-related functions | The student work has the following characteristics:<br>• evidence of simple problem-solving strategies in the context of function problems | The student work has the following characteristic:<br>• evidence of simple mathematical procedures |
| | • identification of assumptions and their associated effects, parameters and/or variables | • identification of assumptions, parameters and/or variables | | | |
| *Communication and Justification* | • appropriate interpretation and use of mathematical terminology in complex non-routine, in life-related functions | • appropriate interpretation and use of mathematical terminology in simple non-routine tasks, in life-related functions | • appropriate interpretation and use of mathematical terminology in simple routine functions | • use of mathematical terminology in simple or routine functions | • use of mathematical terminology |
| | • use of mathematical reasoning to develop logical sequences within a complex response and in life-related functions using everyday and mathematical language | • use of mathematical reasoning to develop logical sequences within a response in simple life-related functions using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine functions using everyday or mathematical language | | |

# Modelling and problem solving
# Mathematics C

## 1. Start with the General Objectives

For Mathematics C, the assessable General Objectives are:

### Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- modify mathematical models as appropriate.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 2. Determine what aspects of the General Objectives you wish to assess.

### Modelling and problem solving

- apply problem-solving strategies and procedures to identify problems to be solved and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- modify mathematical models as appropriate.

### Communication and justification

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

## 3. Refer to the Standards Matrix for the relevant descriptors for the aspects of the General Objectives chosen

| | *Standard A* | *Standard B* | *Standard C* | *Standard D* | *Standard E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple tasks through to routine complex tasks in life-related or abstract situations | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations | The student work has the following characteristics:<br>• evidence of simple problem-solving strategies in the context of problems | The student work has the following characteristic:<br>• evidence of simple mathematical procedures |
| | • identification of assumptions and their associated effects, parameters and/or variables | • identification of assumptions, parameters and/or variables | | | |
| | • use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations | • use of data to synthesise mathematical models in simple situations and generation of data from mathematical models in simple through to complex situations | • use of mathematical models to represent routine, simple situations and generate data | • use of given simple mathematical models to generate data | |
| | • investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems, the strengths and limitations of models, both given and developed | • interpretation of results in the context of simple through to complex problems and mathematical models | • interpretation of results in the context of routine, simple problems | | |
| | • refinement of mathematical models | | | | |

| | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations | The student's work has the following characteristics:<br>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions in simple or routine situations | The student's work has the following characteristics:<br>• use of mathematical terminology, symbols or conventions |
| | • organisation and presentation of information in a variety of representations | • organisation and presentation of information in a variety of representations | • organisation and presentation of information | • presentation of information | • presentation of information |
| | • analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine | • analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine | • translation of information from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language | • use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language | | |
| | • coherent, concise and logical justification of procedures, decisions and results | • coherent and logical justification of procedures, decisions and results | • justification of procedures, decisions or results | | |
| | • justification of the reasonableness of results | | | | |
| | • provision of supporting arguments in the form of proof | | | | |

## 4. Develop an instrument or instruments that will allow students to demonstrate these attributes

**Question 1**

The Millikan oil droop experiment is a famous experiment which was used to determine the charge on an electron. In order to perform the calculations, it is necessary to solve the equation that describes the speed of a small oil drop falling under the action of gravity. This equation is

$$\frac{dv}{dt} = 9.8 - \frac{cv}{d^2}$$

where $c = 3.1 \times 10^{-6}$ ms$^{-1}$ and $d$ = the diameter of the oil drop measured in metres. It is assumed that the oil drop starts from rest.

If the oil drop has a diameter of 1 mm, calculate, correct to 4 decimal places, its velocity after 2 seconds.

**Question 2**

A skyrocket is projected vertically upwards in air with an initial velocity of 20 ms$^{-1}$. If its acceleration, $a$ ms$^{-2}$, at any time $t$ seconds after initial projection is given by $a = -(g + 0.1v^2)$, find the greatest height reached by the skyrocket.

**Question 2 (Modified for evaluation of the validity of an argument)**

A skyrocket is projected vertically upwards in air with an initial velocity of 20 ms$^{-1}$. The acceleration, $a$ ms$^{-2}$, at any time $t$ seconds after initial projection is given by $a = -(g + 0.1v^2)$.

The manufacturer of the skyrocket claims that the rocket will reach a vertical height of at least 25 metres after initial projection. Determine if this claim is valid or not.

Just more English comprehension

**Question 3**

A group of students are modelling a bungy jump. A body of mass 8kg is attached to one end of an elastic string of length 2 metres and modulus of elasticity 60 newtons. The free end of the string is attached 4 metres up the side of a building from the ground, and the mass is allowed to fall from a position adjacent to where the string is attached to the building.

By using a mathematical argument, decide if the string is long enough for the experiment to work.

## 5. Develop an instrument-specific criteria sheet using the chosen attributes of standards matrix as a basis

| | *Standard A* | *Standard B* | *Standard C* | *Standard D* | *Standard E* |
|---|---|---|---|---|---|
| *Modelling and Problem solving* | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret problems to develop responses to non-routine complex tasks in life-related dynamics | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret problems to develop responses to routine complex tasks in life-related dynamics | The student work has the following characteristics:<br>• use of problem-solving strategies to interpret responses to routine, simple problems in life-related dynamics | The student work has the following characteristics:<br>• evidence of simple problem-solving strategies in the context of dynamics | The student work has the following characteristic:<br>• evidence of simple mathematical procedures |
| | • investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of dynamics problems | • interpretation of results in the context of simple through to complex dynamics problems | • interpretation of results in the context of routine, simple dynamics problems | | |
| *Communication and Justification* | • appropriate interpretation and use of mathematical terminology in complex non-routine, in life-related dynamics situations | • appropriate interpretation and use of mathematical terminology in complex routine, in life-related dynamics situations | • appropriate interpretation and use of mathematical terminology in simple routine dynamics situations | • use of mathematical terminology in simple or routine dynamics situations | • use of mathematical terminology |
| | • use of mathematical reasoning to develop logical sequences within a response to complex and in life-related dynamics situations using mathematical language | • use of mathematical reasoning to develop logical sequences within a response in simple life-related dynamics situations using mathematical language | • use of mathematical reasoning to develop sequences within a response in simple routine dynamics situations using mathematical language | | |

# Unpacking the Standards

**The standards descriptors indicate how the various attributes change from one level to another. The key words which can be classified broadly into "doing" or "verb type" words, "nouns" and "adjective" or descriptive words.**

**Within the Mathematics standards there are words which cannot be changed when creating an instrument specific criteria sheet.**

**Words which describe the situation or context can be changed as the situation or context changes.**

**Where multiple descriptors are used, one, some or all of these may be used as appropriate.**

**The highlighted standards which follow attempt to show these words and how they change from one standard to another.**

# 1. Highlighted Standards for Mathematics A

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>accurate use of [redacted] in [redacted] situations | The student's work has the following characteristics:<br>accurate use of [redacted] in [redacted] situations or use of [redacted] in [redacted] situations | The student's work has the following characteristics:<br>use of [redacted] in [redacted] situations | The student's work has the following characteristics:<br>use of [redacted] in [redacted] situations | The student's work has the following characteristics:<br>attempted use of [redacted] in [redacted] situations |
| | application of [redacted] sequences of mathematical procedures in [redacted] situations | application of [redacted] sequences of mathematical procedures in [redacted] situations or [redacted] sequences in [redacted] situations | application of [redacted] sequences of mathematical procedures in [redacted] situations | application of [redacted] mathematical procedures in [redacted] situations | attempted use of [redacted] mathematical procedures in [redacted] situations |
| | appropriate selection and accurate use of technology | appropriate selection and accurate use of technology | selection and use of technology | use of technology | attempted use of technology |

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student's work has the following characteristics:<br>use of [redacted] problems in [redacted] situations | The student's work has the following characteristics:<br>use of [redacted] problems in [redacted] situations | The student's work has the following characteristics:<br>use of [redacted] for problem solving in [redacted] situations | The student's work has the following characteristics:<br>use of [redacted] for problem solving in [redacted] situations | The student's work has the following characteristics:<br>attempted use of [redacted] for problem solving in [redacted] situations |
| | investigation of alternative solutions and/or procedures to [redacted] problems | investigation of alternative solutions and/or procedures to [redacted] problems | | | |
| | informed decisions based on mathematical reasoning in [redacted] through to [redacted] situations | informed decisions based on mathematical reasoning in [redacted] situations | informed decisions based on mathematical reasoning in [redacted] situations | | |
| | reflection on the effectiveness of mathematical models including recognition of the strengths and limitations of the model | recognition of the strengths and limitations of the model in simple situations | | | |

| *Criterion* | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>accurate and appropriate use of mathematical terminology [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>accurate and appropriate use of mathematical terminology [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>appropriate use of mathematical terminology [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology [redacted] conventions in [redacted] situations |
| | organisation and presentation of information in a variety of representations in [redacted] situations | organisation and presentation of information in a variety of representations in [redacted] situations | organisation and presentation of information in a variety of representations in [redacted] situations | presentation of information in [redacted] situations | |
| | analysis and translation of information displayed from one representation to another in [redacted] situations | analysis and translation of information displayed from one representation to another in [redacted] situations | translation of information displayed from one representation to another in [redacted] situations | | |
| | use of mathematical reasoning to develop logical sequences in [redacted] situations using everyday and/or mathematical language | use of mathematical reasoning to develop logical sequences in [redacted] situations using everyday and/or mathematical language | development of logical sequences in [redacted] situations using everyday and/or mathematical language | | |
| | justification of the reasonableness of results obtained through technology or other means | | | | |

## 2. Highlighted Standards for Mathematics B

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] tasks, in life-related [redacted] abstract situations | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] tasks, in life-related [redacted] abstract situations | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] life-related [redacted] abstract situations | The student work has the following characteristics:<br>use of stated rules and procedures in [redacted] situations | The student work has the following characteristics:<br>statements of relevant mathematical facts |
| | application of mathematical definitions, rules and procedures in [redacted] in life-related [redacted] abstract situations | application of mathematical definitions, rules and procedures in [redacted], in either life-related [redacted] abstract situations | application of mathematical definitions, rules and procedures [redacted] life-related [redacted] abstract situations | | |
| | numerical calculations, [redacted] in [redacted] tasks, in life-related [redacted] abstract situations | numerical calculations, [redacted] in [redacted] tasks, in either life-related [redacted] abstract situations | numerical calculations, [redacted] in life-related [redacted] abstract situations | numerical sense, [redacted] in [redacted] tasks | |
| | appropriate selection and accurate use of technology | appropriate selection and accurate use of technology | selection and use of technology | use of technology | use of technology |

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student work has the following characteristics:<br>use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from [redacted] tasks in life-related and abstract situations | The student work has the following characteristics:<br>use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to [redacted] in life-related or abstract situations | The student work has the following characteristics:<br>use of problem-solving strategies to interpret, clarify and develop responses to [redacted] problems in life-related or abstract situations | The student work has the following characteristics:<br>evidence of simple problem-solving strategies in the context of problems | The student work has the following characteristic:<br>evidence of simple mathematical procedures |
| | identification of assumptions and their associated effects, parameters and/or variables | identification of assumptions, parameters and/or variables | | | |
| | use of data to synthesise mathematical models and generation of data from mathematical models in [redacted] situations | use of data to synthesise mathematical models in [redacted] situations and generation of data from mathematical models in [redacted] situations | use of mathematical models to represent [redacted] situations and generate data | use of given simple mathematical models to generate data | |
| | investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems; the strengths and limitations of models, both given and developed | interpretation of results in the context of [redacted] problems and mathematical models | interpretation of results in the context of [redacted] problems | | |

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols and conventions from [redacted] and from [redacted], in life-related [redacted] abstract situations | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols and conventions in [redacted] and from [redacted], in life-related [redacted] abstract situations | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols and conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology, symbols or conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology, symbols or conventions |
| | organisation and presentation of information in a variety of representations | organisation and presentation of information in a [redacted] | organisation and presentation of information | presentation of information | presentation of information |
| | analysis and translation of information from one representation to another in [redacted] situations from [redacted] from [redacted] | analysis and translation of information from one representation to another in [redacted] situations, [redacted] from [redacted] | translation of information from one representation to another in [redacted] situations | | |
| | use of mathematical reasoning to develop coherent, concise and logical sequences within a response from [redacted] situations using everyday [redacted] mathematical language | use of mathematical reasoning to develop coherent and logical sequences within a response in [redacted] situations using everyday [redacted] mathematical language | use of mathematical reasoning to develop sequences within a response in [redacted] situations using everyday [redacted] mathematical language | | |
| | coherent, concise and logical justification of [redacted] decisions [redacted] results | coherent and logical justification of [redacted] decisions [redacted] results | justification of [redacted] [redacted] results | | |
| | justification of the reasonableness of results | | | | |

## 3. Highlighted Standards for Mathematics C

| Criterion | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] tasks, in life-related [redacted] abstract situations | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] tasks in life-related [redacted] abstract situations | The student work has the following characteristics:<br>recall, access, selection of mathematical definitions, rules and procedures in [redacted] life-related [redacted] abstract situations | The student work has the following characteristics:<br>use of stated rules and procedures in [redacted] situations | The student work has the following characteristics:<br>statements of relevant mathematical facts |
| | application of mathematical definitions, rules and procedures in [redacted] tasks, in life-related [redacted] abstract situations | application of mathematical definitions, rules and procedures in [redacted] tasks, in either life-related [redacted] abstract situations | application of mathematical definitions, rules and procedures in [redacted] life-related [redacted] abstract situations | | |
| | numerical calculations, [redacted] in [redacted] tasks, in life-related [redacted] abstract situations | numerical calculations, [redacted] in [redacted] tasks, in either life-related [redacted] abstract situations | numerical sense, [redacted] in [redacted] life-related [redacted] abstract situations | numerical sense, [redacted] in [redacted] tasks | |
| | appropriate selection and accurate use of technology | appropriate selection and accurate use of technology | selection and use of technology | use of technology | use of technology |
| | knowledge of the nature of and use of mathematical proof | | | | |

| | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Modelling and problem solving* | The student work has the following characteristics:<br>use of problem-solving strategies to [redacted] to develop responses from [redacted] tasks in life-related [redacted] abstract situations | The student work has the following characteristics:<br>use of problem-solving strategies to [redacted] to develop responses to [redacted] tasks in life-related [redacted] abstract situations | The student work has the following characteristics:<br>use of problem-solving strategies to [redacted] and develop responses to [redacted] problems in life-related [redacted] abstract situations | The student work has the following characteristics:<br>evidence of simple problem-solving strategies in the context of problems | The student work has the following characteristic:<br>evidence of simple mathematical procedures |
| | identification of assumptions and their associated effects, parameters and/or variables | identification of assumptions, parameters and/or variables | | | |
| | use of data to synthesise mathematical models and generation of data from mathematical models in [redacted] situations | use of data to synthesise mathematical models in [redacted] situations and generation of data from mathematical models in [redacted] situations | use of mathematical models to represent [redacted] situations and generate data | use of given simple mathematical models to generate data | |
| | investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems, the [redacted] | interpretation of results in the context of [redacted] problems and mathematical models | interpretation of results in the context of [redacted] problems | | |
| | refinement of mathematical models | | | | |

| | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|
| *Communication and justification* | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols and conventions from [redacted] and from [redacted], in life-related and abstract situations | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols [redacted] conventions in [redacted] and from [redacted], in life-related or abstract situations | The student's work has the following characteristics:<br>appropriate interpretation and use of mathematical terminology, symbols [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology, symbols [redacted] conventions in [redacted] situations | The student's work has the following characteristics:<br>use of mathematical terminology, symbols [redacted] conventions |
| | organisation and presentation of information in a variety of representations | organisation and presentation of information in a variety of representations | organisation and presentation of information | presentation of information | presentation of information |
| | analysis and translation of information from one representation to another in [redacted] situations from [redacted] and from [redacted] | analysis and translation of information from one representation to another in [redacted] situations, [redacted], and from [redacted] | translation of information from one representation to another in [redacted] situations | | |
| | use of mathematical reasoning to develop coherent, concise and logical sequences within a response from [redacted] and in [redacted] situations using everyday and mathematical language | use of mathematical reasoning to develop coherent and logical sequences within a response in [redacted] and in [redacted] using everyday and mathematical language | use of mathematical reasoning to develop sequences within a response in [redacted] situations using everyday and mathematical language | | |
| | coherent, concise and logical justification of procedures, decisions and results | coherent and logical justification of [redacted] | justification of procedures [redacted] | | |
| | justification of the reasonableness of results | | | | |
| | provision of supporting arguments in the form of proof | | | | |

# Developing a Profile

**This example of a way to develop a profile suggests a different method of data collection. It is hoped that this will promote discussion about the effectiveness of the ways in which data about students' work is gathered.**

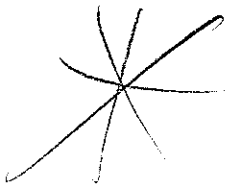
**This particular format allows for more than just a means of aggregating information.**

**At any point in time, an "on balanced" judgment can be made on overall performance.**

**Overall student performance within a particular criterion can be assessed at a glance. Areas of concern, where performance within certain attributes is not consistent with others, can be easily identified. The format shows this to students.**

**The completeness of the assessment package, ie all individual dot points of all three criteria are included, can be easily determined. If anything is missing at a particular point in time, it can be identified and included in later assessment.**

**The balance of the overall package can also be easily identified and adjustments can be made accordingly.**



# Developing a Profile

1. The Blank Profile
2. Criteria Sheet for Instrument 1
3. Profile after Instrument 1
4. Criteria Sheet for Instrument 1
5. Cumulative Profile after Instrument 2
6. Criteria Sheet for Instrument 3
7. Cumulative Profile after Instrument 3
8. Criteria Sheet for Instrument 4
9. Cumulative Profile after Instrument 4

## The Blank Profile

| *Criterion* | | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|---|
| *Knowledge and procedures* | ***K1*** | | | | | |
| | ***K2*** | | | | | |
| | ***K3*** | | | | | |
| *Modelling and problem solving* | ***M1*** | | | | | |
| | ***M2*** | | | | | |
| | ***M3*** | | | | | |
| | ***M4*** | | | | | |
| *Communication andd justification* | ***C1*** | | | | | |
| | ***C2*** | | | | | |
| | ***C3*** | | | | | |
| | ***C4*** | | | | | |
| | ***C5*** | | | | | |

## Criteria Sheet for Instrument 1

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>• accurate use of rules and formulas in complex situations | The student's work has the following characteristics:<br>• accurate use of rules and formulas in simple situations | The student's work has the following characteristics:<br>• use of rules and formulas in simple routine situations | The student's work has the following characteristics:<br>• use of given rules and formulas in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given rules and formulas in simple rehearsed situations |
| | • appropriate selection and accurate use of technology | • appropriate selection and accurate use of technology | • selection and use of technology | • use of technology | • attempted use of technology |
| *Modelling and problem solving* | • use of strategies to model and solve problems in simple non-routine situations | • use of strategies to model and solve problems in routine situations | • use of familiar strategies for problem solving in simple routine situations | • use of given strategies for problem solving in simple rehearsed situations | • attempted use of given strategies for problem solving in well-rehearsed situations |
| | • informed decisions based on mathematical reasoning in complex routine situations | • informed decisions based on mathematical reasoning in routine situations | • informed decisions based on mathematical reasoning in simple routine situations | | |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in complex routine situations | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • appropriate use of mathematical terminology and conventions in simple routine situations | • use of mathematical terminology and conventions in simple rehearsed situations | • use of mathematical terminology or conventions in simple rehearsed situations |
| | • organisation and presentation of information in a variety of representations in complex routine situations | • organisation and presentation of information in a variety of representations in simple non-routine situations | • organisation and presentation of information in a variety of representations in simple routine situations | • presentation of information in simple rehearsed situations | |
| | • use of mathematical reasoning to develop logical sequences in complex routine situations using everyday language | • use of mathematical reasoning to develop logical sequences in simple non-routine situations using everyday language | • development of logical sequences in simple routine situations using everyday language | | |
| | • justification of the reasonableness of results obtained through technology or other means | | | | |

## Profile after Instrument 1

| Criterion | | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|---|
| Knowledge and procedures | K1 | | X | | | |
| | K2 | | | | | |
| | K3 | X | | | | |
| Modelling and problem solving | M1 | | | X | | |
| | M2 | | | | | |
| | M3 | | | X | | |
| | M4 | | | | | |
| Communication and justification | C1 | | X | | | |
| | C2 | | X | | | |
| | C3 | | | | | |
| | C4 | | | X | | |
| | C5 | | | | | |

## Criteria Sheet for Instrument 2

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>• accurate use of rules and formulas in complex situations | The student's work has the following characteristics:<br>• use of rules and formulas in complex situations | The student's work has the following characteristics:<br>• use of rules and formulas in simple routine situations | The student's work has the following characteristics:<br>• use of given rules and formulas in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given rules and formulas in simple rehearsed situations |
| | • application of complex sequences of mathematical procedures in non-routine situations | • application of complex sequences of mathematical procedures in routine situations | • application of simple sequences of mathematical procedures in routine situations | • application of simple mathematical procedures in simple rehearsed situations | • attempted use of simple mathematical procedures in simple rehearsed situations |
| *Modelling and problem solving* | • use of strategies to model and solve problems in simple non-routine situations | • use of strategies to model and solve problems in routine situations | • use of familiar strategies for problem solving in simple routine situations | • use of given strategies for problem solving in simple rehearsed situations | • attempted use of given strategies for problem solving in well-rehearsed situations |
| | • investigation of alternative solutions and/or procedures to simple non-routine problems | • investigation of alternative solutions and/or procedures to routine problems | | | |
| | • reflection on the effectiveness of mathematical models including recognition of the strengths and limitations of the model | • recognition of the strengths and limitations of the model in simple situations | | | |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • appropriate use of mathematical terminology and conventions in simple routine situations | • use of mathematical terminology and conventions in simple rehearsed situations | • use of mathematical terminology or conventions in simple rehearsed situations |
| | • analysis and translation of information displayed from one representation to another in simple non-routine situations | • analysis and translation of information displayed from one representation to another in simple non-routine situations | • translation of information displayed from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop logical sequences in simple non-routine situations using mathematical language | • use of mathematical reasoning to develop logical sequences in simple non-routine situations using mathematical language | • development of logical sequences in simple routine situations using mathematical language | | |

## Profile after Instrument 2

| Criterion | | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|---|
| Knowledge and procedures | K1 | | X X | | | |
| | K2 | | X | | | |
| | K3 | X | | | | |
| Modelling and problem solving | M1 | | X | X | | |
| | M2 | | X | | | |
| | M3 | | | X | | |
| | M4 | X | | | | |
| Communication and justification | C1 | | X X | | | |
| | C2 | | X | | | |
| | C3 | | X | | | |
| | C4 | X | | X | | |
| | C5 | | | | | |

# Criteria Sheet for Instrument 3

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>• application of simple through to complex sequences of mathematical procedures in non-routine situations | The student's work has the following characteristics:<br>• application of complex sequences of mathematical procedures in routine situations | The student's work has the following characteristics:<br>• application of simple sequences of mathematical procedures in routine situations | The student's work has the following characteristics:<br>• application of simple mathematical procedures in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of simple mathematical procedures in simple rehearsed situations |
| | • appropriate selection and accurate use of technology | • appropriate selection and accurate use of technology | • selection and use of technology | • use of technology | • attempted use of technology |
| *Modelling and problem solving* | • use of strategies to model and solve problems in simple non-routine situations | • use of strategies to model and solve problems in routine situations | • use of familiar strategies for problem solving in simple routine situations | • use of given strategies for problem solving in simple rehearsed situations | • attempted use of given strategies for problem solving in well-rehearsed situations |
| | • investigation of alternative solutions and/or procedures to simple non-routine problems | • investigation of alternative solutions and/or procedures to routine problems | | | |
| | • informed decisions based on mathematical reasoning in simple non-routine situations | • informed decisions based on mathematical reasoning in routine situations | • informed decisions based on mathematical reasoning in simple routine situations | | |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • accurate and appropriate use of mathematical terminology and conventions in simple non-routine situations | • appropriate use of mathematical terminology and conventions in simple routine situations | • use of mathematical terminology and conventions in simple rehearsed situations | • use of mathematical terminology or conventions in simple rehearsed situations |
| | • organisation and presentation of information in a variety of representations in simple non-routine situations | • organisation and presentation of information in a variety of representations in simple non-routine situations | • organisation and presentation of information in a variety of representations in simple routine situations | • presentation of information in simple rehearsed situations | |
| | • justification of the reasonableness of results obtained through technology or other means | | | | |

## Profile after Instrument 3

| Criterion | | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|---|
| *Knowledge and procedures* | ***K1*** | | ***X X*** | | | |
| | ***K2*** | | ***XX*** | | | |
| | ***K3*** | ***X X*** | | | | |
| *Modelling and problem solving* | ***M1*** | | ***XX*** | ***X*** | | |
| | ***M2*** | | ***XX*** | | | |
| | ***M3*** | | | ***X X*** | | |
| | ***M4*** | ***X*** | | | | |
| *Communication and justification* | ***C1*** | | ***X XX*** | | | |
| | ***C2*** | | ***X*** | ***X*** | | |
| | ***C3*** | | ***X*** | | | |
| | ***C4*** | ***X*** | | ***X*** | | |
| | ***C5*** | | | | | |

# Criteria Sheet for Instrument 4

| *Criterion* | Standard *A* | Standard *B* | Standard *C* | Standard *D* | Standard *E* |
|---|---|---|---|---|---|
| *Knowledge and procedures* | The student's work has the following characteristics:<br>• accurate use of rules and formulas in complex situations | The student's work has the following characteristics:<br>• accurate use of rules and formulas in simple situations | The student's work has the following characteristics:<br>• use of rules and formulas in simple routine situations | The student's work has the following characteristics:<br>• use of given rules and formulas in simple rehearsed situations | The student's work has the following characteristics:<br>• attempted use of given rules and formulas in simple rehearsed situations |
| | • application of complex sequences of mathematical procedures in non-routine situations | • application of simple sequences of mathematical procedures in non-routine situations | • application of simple sequences of mathematical procedures in routine situations | • application of simple mathematical procedures in simple rehearsed situations | • attempted use of simple mathematical procedures in simple rehearsed situations |
| *Modelling and problem solving* | • use of strategies to model and solve problems in complex routine situations | • use of strategies to model and solve problems in routine situations | • use of familiar strategies for problem solving in simple routine situations | • use of given strategies for problem solving in simple rehearsed situations | • attempted use of given strategies for problem solving in well-rehearsed situations |
| | • investigation of alternative solutions and/or procedures to complex routine problems | • investigation of alternative solutions and/or procedures to routine problems | | | |
| | • reflection on the effectiveness of mathematical models including recognition of the strengths and limitations of the model | • recognition of the strengths and limitations of the model in simple situations | | | |
| *Communication and justification* | • accurate and appropriate use of mathematical terminology and conventions in complex routine situations | • accurate and appropriate use of mathematical terminology and conventions in complex routine situations | • appropriate use of mathematical terminology and conventions in simple routine situations | • use of mathematical terminology and conventions in simple rehearsed situations | • use of mathematical terminology or conventions in simple rehearsed situations |
| | • analysis and translation of information displayed from one representation to another in complex routine situations | • analysis and translation of information displayed from one representation to another in simple routine situations | • translation of information displayed from one representation to another in simple routine situations | | |
| | • use of mathematical reasoning to develop logical sequences in complex routine situations using everyday language | • use of mathematical reasoning to develop logical sequences in complex routine situations using everyday l language | • development of logical sequences in simple routine situations using everyday language | | |

## Profile after Instrument 4

| Criterion | | Standard A | Standard B | Standard C | Standard D | Standard E |
|---|---|---|---|---|---|---|
| Knowledge and procedures | K1 | | X XX | | | |
| | K2 | | X XX | | | |
| | K3 | X X | | | | |
| Modelling and problem solving | M1 | | XX | XX | | |
| | M2 | | XXX | | | |
| | M3 | | | X X | | |
| | M4 | X | X | | | |
| Communication and justification | C1 | X | X XX | | | |
| | C2 | | X | X | | |
| | C3 | | X X | | | |
| | C4 | X | X | X | | |
| | C5 | | | | | |

## Overall Grade: B

**Finally, something that you may find useful in your schools.**

# Questions for an EMPS

## Question 1

An oil refinery has a tall chimney stack from which a pollutant gas is emitted at the rate of 2000 kilograms per day. Engineers at the refinery have developed new technology which allows the reduction of these emissions to a minimum of 250 kilograms per day. To achieve this minimum level, the emissions are to be reduced by a constant amount each day until the minimum emission of 250 kilograms is reached.

Calculate the constant amount by which the emissions are reduced each day if the installation is completed by the end of the $10^{th}$ day, and from the $11^{th}$ day the emission will be 250 kilograms per day.

Determine the total emissions during the 10 days of installation.

Due to fortuitous circumstances, the installation is completed by the end of the $7^{th}$ day, so that from the $8^{th}$ day the emission will be 250 kilograms per day. Calculate the new constant amount by which the emissions must be reduced each day during this installation period.

By using a different installation method, the emissions will be reduced by a constant percentage each day until the minimum emission of 250 kilograms per day is reached. The engineers have determined that the constant percentage to be used is 25.7%.

Determine, using the constant percentage method, on which day the daily emissions (within 1 kilogram) will reach the minimum of 250 kilograms per day.

By comparing the constant percentage method to the constant amount method, determine the difference, to the nearest kilogram, in the total emissions during the first ten days of operation.

If it is not desired that the minimum of 250 kilograms of emissions be reached until the $10^{th}$ day, what constant percentage must the emissions be reduced by each day to reach this target?

New government legislation requires that the minimum emissions for this particular installation be 200 kilograms per day. By considering both the constant amount and the constant percentage methods of reduction, determine the effects that the new legislation would have on the calculations of the engineers.

**Question 2**

A micro-organism under investigation has an initial population of 10 000, which increases by 2000 microbes each hour. A group of scientists is investigating the control of this particular organism.

By completing the following table

| Hour | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Population of microbes | | | | |

construct a graph showing the population growth of the microbes, and determine what the population would be after 8 hours.

The scientists introduce a control organism after 8 hours that reduces the population of the microbes by 12% each hour.

By completing the following table

| Hours after introduction of control | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Population of microbes | | | | |

determine how many microbes are present 5 hours after the introduction of the control.

The organism is considered to be under control when the population is reduced to one-quarter of the population from the time the control organism was introduced.

Determine, to the nearest hour, how long after the introduction of the control organism when the micro-organism is considered to be under control.

To establish the effect of the response time, the experiment is repeated, but this time the original microbe is left for 10 hours before the control is introduced. Determine the effect of this variation on the time to control the micro-organism.

Determine if there is an optimal time for introduction of the control in relation to the time when the micro-organism is considered to be under control.